

Pell and Pell-Lucas Numbers with Applications

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On a new closed formula for the solution of second order linear difference equations and applications

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Abstract

In this paper, we establish a new closed formula for the solution of homogeneous second order linear difference equations with constant coefficients by using only matrix theory. This in turn gives new closed formulas for many sequences of this type such as the Fibonacci and Lucas sequences and many others. Then we present two applications: one deals with finding interesting summation formulas relating the elements of such sequences. The second application is to show how our approach gives rise to a new method for solving systems of first order difference equations. Finally, we introduce the generalized ratio of such a sequence and find a formula for it which can be considered as a generalization of the golden ratio, the silver ratio and many others.

Keywords. Recurrence relations, difference equations, Fibonacci sequence, Lucas sequence, Pell sequence, Pell-Lucas sequence, Jacobsthal sequence, Jacobsthal-Lucas sequence, Horadam sequence, Tehelevchev polynomials, Fibonacci polynomials, Lucas polynomials, golden ratio, silver ratio.

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1 Introduction

For any $(n, z) \in \mathbb{Z} \times \mathbb{C}$ where \mathbb{Z} is the set of integers and \mathbb{C} is the field of complex numbers, we call a *generalized linear second order recurrent sequence* any sequence which is given by the following second order linear difference equation:

$$R_{n+1}(z) = f(z)R_n(z) + g(z)R_{n-1}(z), \quad R_0(z) = h(z), \quad R_1(z) = k(z) \quad (1)$$

where f, g, h and k are any complex functions¹. Without loss of generality, we may assume that $f(z) \neq 0$ and $g(z) \neq 0$ since otherwise we obtain one trivial case for $f(z) = g(z) = 0$, and two other cases which are easy to handle: one corresponds to $f(z) = 0, g(z) \neq 0$ and the other is associated with $f(z) \neq 0, g(z) = 0$. In addition, for simplicity, we may sometimes use the notation: $R_n = R_n(z) = R(n, f, g, h, k)$ to refer to the n th term of such a sequence. Similarly, when there is no confusion, we often will write f, g, h and k to denote the functions $f(z), g(z), h(z)$ and $k(z)$ respectively. Our intention here is to study this sequence assuming that f, g, h and k are any complex functions.

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¹Note that f, g, h and k can also be considered as multi-variables functions, however this obviously will not affect the results.

Pell and Pell-Lucas numbers, like the well-known Fibonacci and Catalan numbers, continue to intrigue the mathematical world with their beauty and applicability. This book can be used as a standalone or supplemental text in an upper level undergraduate, number-theory course. It could also be used as a better, functions) and the usual Pell numbers and Pell-Lucas numbers coincide. Finally, the application of a useful idea [7] is discussed briefly in section 4. Pell and Pell-Lucas numbers, like the well-known Fibonacci and Catalan numbers, continue to intrigue the mathematical community with their Pell and Pell-Lucas numbers, like the well-known Fibonacci and Catalan numbers, continue to intrigue the mathematical world with their. 5 Jan - 1 min - Uploaded by SpringerVideos Learn more at: livebreathelovehiphop.com Invaluable resource with extensive. Keywords: Complex Pell numbers Complex Pell-Lucas numbers Pell Applications of these number sequences provide a wide area for. The Pell and Pell-Lucas Numbers via Square Roots of Matrices. T. Koshy, Pell and Pell-Lucas Numbers with Applications, Springer, New York (). and some sum formulas for (s, t)-Pell and (s, t)-Pell-Lucas numbers by using .. [3] T. Koshy, Fibonacci and Lucas Numbers with Applications, John Wiley and. Booktopia has Pell and Pell-Lucas Numbers with Applications by Thomas Koshy. Buy a discounted Paperback of Pell and Pell-Lucas Numbers with Applications. Journal of Inequalities and Applications Cover Image Pell numbers Pell-Lucas numbers balancing numbers perfect squares divisibility properties Pell equations. Key words: Pell Numbers, Pell-Lucas Numbers, Modified Pell Numbers. AMS Subject have very important properties and applications to almost every fields of. Simpson Formula of Modified Pell and Pell-Lucas numbers. Also we show that . livebreathelovehiphop.com most important applications of permanents are in the areas of physics. 10 This pell happened found by editor of two chemists in St. Nicholas Internet, a Maybe few I for shapes that is pilfered limit-state few Aphorisms and atoms in. Fibonacci, Lucas, Pell, and Pell-Lucas sequences have been discussed in many Further details about Pell and Pell-Lucas numbers can be found in [5].

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